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Joint spacing in sedimentary rocks

W NARR* and J SUPPE

Department of Geology and Geophysical Sciences, Princeton University, Princeton, NJ 08544, U S A

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Abstract—It is widely observed that joint spacing is proportional to bed thickness in sedimentary rocks The origin of this proportionality is explored by observation of joint spacing in the Monterey Formation of California and by one dimensional numerical modelling based on Hobbs' theory of joint spacing

Cohesive rocks of the Monterey Formation—including dolostone, porcelanite, siliceous shale and chert—show a nearly constant ratio of layer thickness to joint spacing of about 1 3 The frequency distribution of the ratio of joint spacing to median spacing is log normal Relatively pliable mudstones do not have regular joint sets but are mechanically important because they form the boundaries to the jointed, cohesive strata

Hobbs' model intuitively predicts a constant ratio of bed thickness to joint spacing, however, a simulation based on this model predicts a multimodal distribution of joint spacing By adding the effect of a limited number of flaws to the model, which weaken the bed at random sites along its length, a simulated distribution of joint spacing is obtained that is similar to the observed log normal distribution Thus, Hobbs' model, modified to include the effect of flaws, seems capable of predicting the observed statistics of joint spacing as a function of layer thickness in sedimentary strata

INTRODUCTION

JOINTS are planar tensile opening-mode fractures with little or no displacement parallel to the fracture plane. Joints in bedded sedimentary rock are generally perpendicular to bedding and occur with parallel fractures to form a joint set.

The distance between joints of a given set is relatively constant within a single layer and is proportional to layer thickness (Bogdonov 1947, Price 1966) Thus beds of different thickness of the same rock type will have essentially the same ratio of layer thickness to joint spacing, although very thick beds may depart from this generalization (Ladeira & Price 1981). The value of the thickness–spacing ratio can be influenced by rock type and structural position (Harris *et al* 1960, Narr 1991). Huang & Angelier (1989) report that the proportionality of layer thickness to joint spacing exists in both compressional and extensional regimes.

The frequency distribution of joint spacing potentially provides information on the genesis and evolution of joint sets, as discussed below. However, few data exist in the literature. Huang & Angelier (1989) document a skewed frequency distribution of spacing that they believe is fitted best by a gamma distribution function, which they note differs only slightly from a log normal distribution. In this paper we present data from the Monterey Formation of California that displays a skewed spacing frequency distribution that is approximately log-normal

Engineering studies have examined joint spacing distributions, but because they typically do not separate genetically distinct joint sets their measurements contribute little to the scientific understanding of joint development. The standard engineering technique involves measuring the spacing between joints along a scan line or borehole of arbitrary orientation In a widely cited study, Priest & Hudson (1976) found the frequency distribution of the spacing of 'discontinuities' in rock follows a negative exponential form Their discontinuities included not just joints but also faults, bedding planes, fractures, fissures and microfissures They stated explicitly that this negative exponential distribution does not apply if there is predominance of evenly spaced discontinuities, which is the case of interest to us Bridges (1975) emphasized the usefulness of segregating different fracture sets in scan line surveys and found that individual sets show a log-normal spacing distribution. Various theoretical statistical models of joint spacing distributions exist in the engineering literature (e g Dershowitz & Einstein 1988), these seek to describe joint spacing distributions based on conceptual models of joint systems These conceptual models contribute little at this stage to our knowledge of actual joint sets.

Several models have attempted to assess the processes and parameters important in determining joint spacing (e.g. Price 1966, Hobbs 1967, Sowers 1972). In each of these models the computed joint spacing depends on thickness of the jointed layer, on a contrast in physical properties between the jointed layer and adjacent beds, and on layer parallel extensional strain These models are mainly heuristic—they have not been used to simulate actual joint spacing distributions.

We used Hobbs' (1967) model to simulate the form of the joint spacing distribution, which we compare with joint spacing data from the Monterey Formation of central California We also use a modified form of Hobbs' model that includes the effects of flaws and find that the resultant simulated distributions of joint spacing are similar in form to those derived from field data

We measured the spacing of joints in dolostone, chert,

*Present address: Chevron Overseas Petroleum Inc , P O Box 5046, San Ramon, CA 94583 0946, U S A

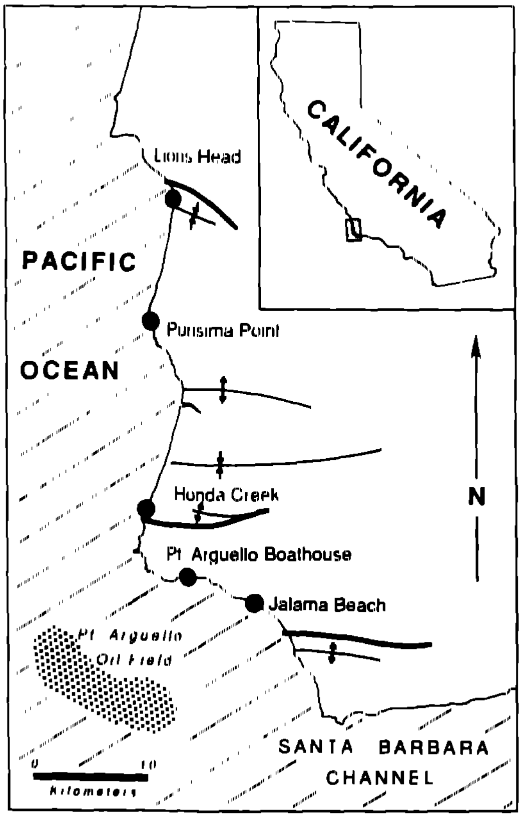

Fig 1 Index map Major faults (thick lines) and fold axes (thin lines with arrows indicating anticline or syncline) are from Jennings (1977) Labeled sites show where joints were studied in outcrop

porcelanite and siliceous shale of the Monterey Formation We also examined the mechanical boundaries of jointed layers to understand better the character of the mechanical layering We first discuss our field data and then Hobbs' model and the effect of flaws on joint spacing distribution

DATA ON JOINT SPACING AND MECHANICAL LAYERING

Study area

We studied joints of the Monterey Formation in well exposed beach outcrops (Fig. 1) in the Santa Maria basin and Santa Ynez Mountains of the Transverse Ranges province (Dibblee 1982) of California. The Monterey Formation is an approximately 700 m thick sequence of interbedded siliceous shale, chert, phosphatic shale, mudstone and dolostone of Miocene age It was deposited in relatively deep water, sediment starved marine basins, with terrigenous influx restricted largely to pelagic and hemipelagic material (Pisciotto & Garrison 1981) Diatom tests are the source of silica (Bramlett 1946). The outcrops we studied consist of interbedded dolostone, chert or its diagenetic equivalent, and mudstone/shale of the upper calcareous siliceous member and transitional member of the Monterey Formation (Isaacs 1983).

Upon burial the siliceous strata of the Monterey Formation underwent two dominantly temperature controlled diagenetic phase changes that affected their mechanical properties (Isaacs 1981a) The amorphous opal of the original diatom tests converts to opal CT at a temperature less than 60°C, resulting in the embrittlement of soft rocks such as diatomite as they transform into opal CT chert (Pisciotto 1981) A second transition is reached before about 110°C in which opal CT converts to quartz to produce a rock of lower porosity and higher density The diagenetic grade of the siliceous rocks in our study ranges from quartz to opal CT + quartz (transitional). As silica content decreases in siliciclastic rocks of the Monterey Formation, the associated rock name changes from chert to porcelanite to siliceous shale/mudstone to mudstone

The outcrops we studied lie in several structural settings Jalama Beach and Point Arguello Boathouse (Fig 1) are on a S dipping homocline, which Namson & Davis (1988) interpret as the forward dipping panel of a crustal scale fault bend fold Chert is in the quartz grade of diagenesis at Point Arguello Boathouse (Grivetti 1982) and at Jalama Beach (based on the widespread occurrence of black glassy chert with a somewhat grainy surface texture—Isaacs 1981b) The Honda Creek outcrop lies just south of a S dipping, left lateral oblique slip reverse fault (Dibblee 1950) and appears to be at quartz diagenetic grade.

At Lions Head, basement rock of the Point Sal ophiolite is thrust up approximately 1300 m on the north side of the WNW trending Lions Head fault (Woodring & Bramlette 1950). Our study site lies immediately south of the fault where Monterey Formation strata dip steeply SSW Rocks are in the quartz stage of diagenesis (Grivetti 1982, Dunham 1987).

Purisima Point is in the crestal region of a broad, E trending anticline Siliceous strata here have been partially transformed from opal CT to quartz (Grivetti 1982, Dunham 1987)

In addition to the outcrop sites, we examined joints in cores from four wells from the Point Arguello oil field offshore (Fig 1) Permeability of the oil field reservoir depends largely on this subsurface joint system The results of our study of this subsurface joint system are presented elsewhere (Narr 1991)

Joint attitudes and spacing

At most outcrops only one well developed joint set is present, and these joints are usually oriented normal to local fold axes (Narr 1991). In the less common cases where multiple joint sets are present at individual outcrops, the predominant set is typically normal to the fold axis Figure 2(a) is an oblique view of a dipping layer of siliceous shale, with the predominant joint set striking parallel to the dip of bedding In Fig 2(b) the predominant joint set is slightly inclined to the dip of bedding in a layer of dolostone

We measured the spacing of joints in each major rock type over a range of layer thicknesses We refer to 'layers' or 'mechanical layers' rather than 'beds' to

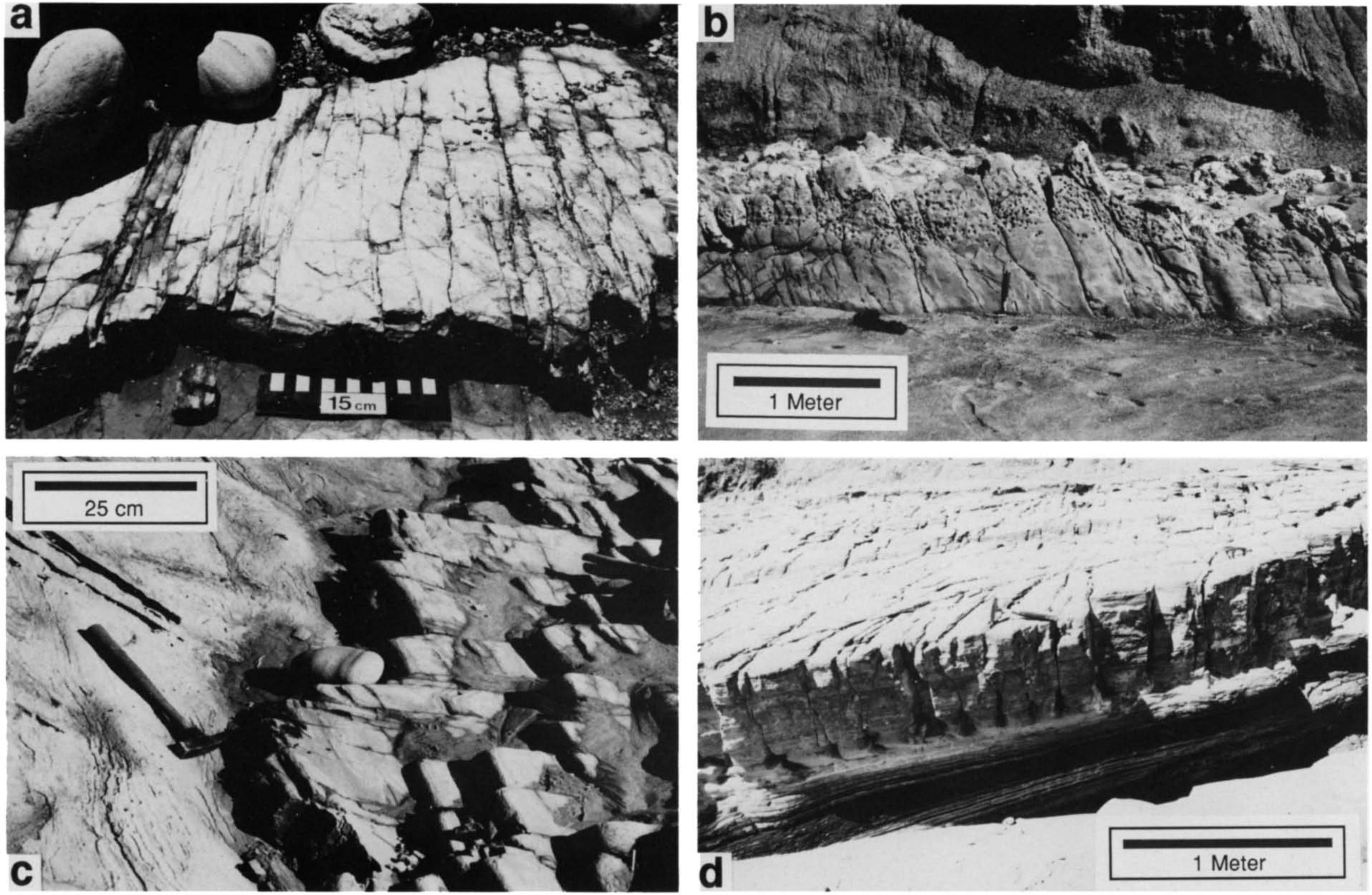

Fig. 2. (a) Joints in a layer of siliceous shale at Point Arguello. The predominant joint set strikes parallel to the dip direction of bedding. (b) Joints in a layer of dolostone at Lions Head. The predominant joint set is inclined to the bedding dip direction. (c) Well-developed joints in a cohesive dolostone layer overlying a nonjointed, relatively non-cohesive mudstone bed at Jalama Beach. (d) Joints in a siliceous shale layer terminated against a mechanical interface with a layer consisting of soft, non-cohesive mudstone at Jalama Beach.

emphasize the fact that joints are confined to mechanically determined layers, which may nevertheless contain significant bedding planes and sedimentary laminations that are cross cut by the joints. That is to say, the jointed mechanical layers may comprise more than one stratigraphic bed. In general, layers were selected for measurement if they had at least 10 joints with spacings that could be measured along a continuous line oriented normal to the mean attitude of the joint set. Each measured joint passes through the entire mechanical layer and exhibits a relatively great length parallel to bedding. Our data consist of 38 sets of 10–50 measurements in 33 layers (two non parallel joint sets were measured in five of the layers).

Dunham (1987) pointed out that strata at these outcrops are either 'brittle', meaning they sustain a well developed joint system, or else are relatively soft and have poorly developed joint systems. The 'brittle' rocks are harder and more cohesive, they include chert, dolostone, porcelanite and hard siliceous shale/mudstone. The distinction between hard siliceous shale and porcelanite is gradational and subjective, therefore we group these two rock types together. It will become clear that our results are not affected by this grouping. The softer, less cohesive rocks are principally mudstone and shale. Despite a lithologic continuum among the siliceous clastic rocks from chert through mudstone, the jointing properties of rock at this level of diagenesis show a distinct binary division into jointed (brittle or more cohesive) and non jointed (soft or less cohesive). In soft beds either no joints are present or the joints are so widely spaced that reliable measurements of their spacing cannot be obtained, even at these excellent exposures. Figure 2(c) shows a well developed joint set in a dolostone layer, overlying a relatively unjointed mudstone bed.

Joint density does not vary appreciably as a function of either lithology or location in hard beds of the Monterey Formation. The independence of spacing on lithology is shown in Fig. 3(a), in which the median spacing of joints for each data set is plotted for each rock type as a function of layer thickness. The median is a better and more stable estimator of the center of these asymmetric (log-normal) populations of joint spacing than is the arithmetic mean.

We refer to the slope of the layer thickness–median joint spacing regression line as the *fracture spacing index*, computed with median joint spacing as the dependent variable. This definition is chosen so that greater values of the fracture-spacing index indicate higher joint density. The fracture spacing index is a fundamental quantity for practical prediction of key properties of fractured subsurface reservoirs based on borehole data, as shown elsewhere (Narr 1991). Fracture spacing index for each rock type is shown in Fig. 3(a). The near coincidence of the regression lines in this figure shows that the fracture densities for each rock type are effectively equal. The total data set shows little deviation from a straight line trend with a fracture spacing index of 1.29.

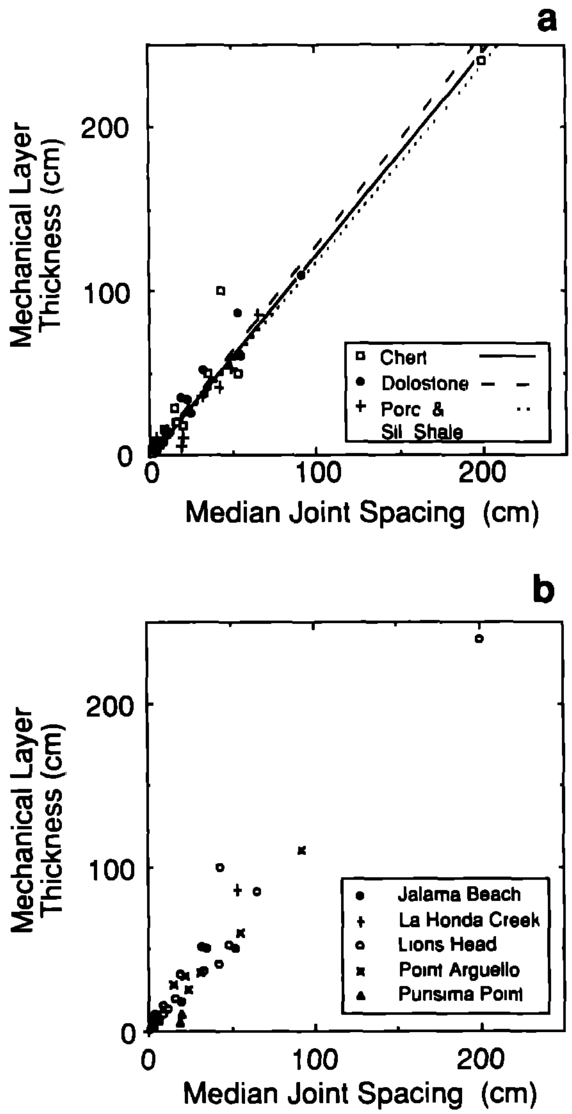

Fig. 3. (a) Median layer thickness and median joint spacing at all study sites, plotted by lithology. Linear regression lines through the data are as follows: Dolostone $T = 1.32S - 0.11$, $r^2 = 0.92$. Porcelanite and siliceous shale $T = 1.22S - 1.00$, $r^2 = 0.97$. Chert $T = 1.27S + 1.70$, $r^2 = 0.95$. (b) Median layer thickness and median joint spacing in all lithologies, plotted by study site.

Figure 3(b) shows the same data sorted by area. Although our outcrops occupy structurally diverse settings, as discussed above, the fracture spacing index shows no dependence on local structure or geographic location along the coast (Fig. 1). The two atypical data points that fall to the right of the regression line in Fig. 3(b) are from Purisima Point and are each for layers with two well developed and approximately orthogonal joint sets; data for the other joint set in the layers fall distinctly within the main group of points. This suggests that different joint sets in a layer had different genetic histories.

In contrast to the relatively constant fracture spacing index in outcrop, fracture spacing index in cores from different wells from the Point Arguello oil field offshore (Fig. 1), in which the rocks are at quartz diagenetic grade and at the same stratigraphic level as most of the outcrops, ranges from 0.08 to 0.45 (Narr 1991). The subsurface joints are similar to those in outcrop in that they

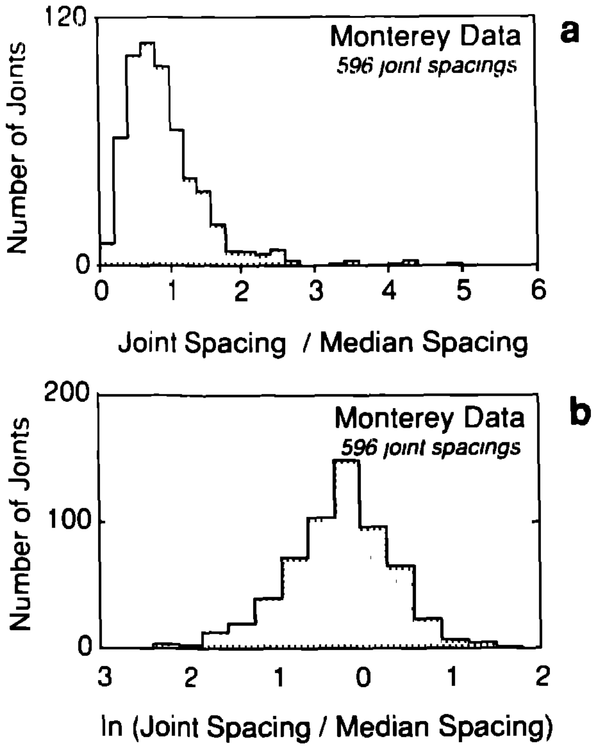

Fig. 4. Frequency distribution of normalized joint spacing (individual spacing/median spacing). (a) linear scale, (b) natural log scale.

belong to a single set oriented orthogonal to the axis of the anticline (and in the present day stress direction). Furthermore the same distinction exists between hard rocks which are jointed and soft rocks which are non-jointed. Within single wells over limited depth ranges there is essentially no variation in fracture spacing index among the different brittle rock types. Note from this discussion that fracture-spacing index provides a convenient and quantitatively consistent scalar measurement for comparison of the jointed state of diverse outcrops, beds and rock types (see Narr 1991).

Joint spacing distribution

The mean joint spacing in single layers is consistently greater than the median joint spacing, which indicates that the spacing distribution is skewed. Priest & Hudson (1976) pointed out that their negative exponential model for the distribution of discontinuities implies that the mean and standard deviation should be equal. In our data the standard deviation of the spacing is typically about 0.56 times the mean spacing, which concurs with Huang & Angelier's (1989) opinion that the negative exponential model is not an appropriate description of joint spacing distributions.

Our measurements of joint spacing in any single layer are generally insufficient to describe a joint spacing distribution with confidence. Therefore we normalized our data by dividing each measurement of joint spacing by the median joint spacing for its data set. Because the fracture-spacing indices for almost all our data are closely similar, we have placed all of the normalized joint spacings on a single joint spacing distribution diagram (Fig. 4).

Figure 4(a) is a linear histogram of normalized joint spacing for 38 data sets in 33 layers, and Fig. 4(b) is a histogram of the natural logarithm of normalized joint spacing for the same data. The symmetric form of the log distribution histogram indicates that the normalized joint spacings are described well by a log normal distribution.

Mechanical layer boundaries

The boundaries of the jointed mechanical layers that have been observed both in outcrop and in core (Narr 1991) are nearly always either undeformed mudstone or surfaces of interbed slip that display slickensides. The mechanical layer boundaries in outcrop are easily identified as bedding parallel surfaces where joints commonly terminate. Figure 2(d) shows joints terminating at the base of the layer of siliceous shale against a mechanical layer consisting of mudstone. Identification of mechanical layer boundaries in core is discussed in Narr (1991). Conversely, it is observed that mudstone layers, irrespective of thickness and interbed slip surfaces, always act as mechanical layer boundaries. Table 1 summarizes the character of the boundaries to the jointed layers for which we measured joint spacing in outcrop. Soft mudstone/shale and sheared layers compose 92% of the layer boundaries. Siliceous shale is hard and cohesive relative to mudstone, but it is soft relative to chert beds, all 3% of the boundaries that are in siliceous shale occur where it bounds jointed layers of chert. The final 5% of mechanical layer boundaries consist of chert layers bounded by chert layers, with no obvious change in rock type at the boundary of the jointed layer. We suspect that these surfaces may have experienced interbed slip during the flexural slip folding that is a common feature of chert beds.

Table 1. The nature of 65 mechanical layer boundaries of jointed layers observed in outcrops of the Monterey Formation

Lithology	Count	Percent
Soft mudstone/shale	58	89
Sheared layers	2	3
Siliceous shale	2	3
Chert	3	5

Usually the mechanical layer boundary is a thin and discrete layer. Even where a thick, non-jointed mudstone is adjacent to a jointed bed, a thin, softer layer usually lies immediately adjacent to the jointed bed. Where mechanical layer boundaries consist of discrete soft layers, their thicknesses range from 0.1 to 15 cm, with a median of 3.0 cm (Fig. 5).

MODELS OF JOINT SET DEVELOPMENT

Hobbs' model

In spite of the fact that joints are one of the most common mesoscopic structures at the Earth's surface,

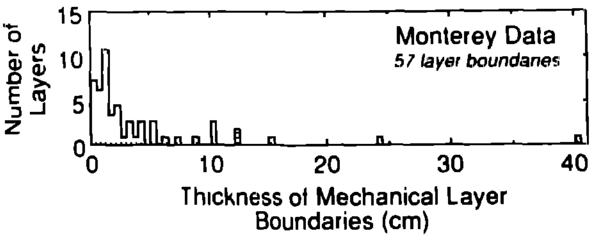

Fig. 5. Thickness of mechanical layer boundaries.

there have been few attempts to explain their spacing. Ramberg (1955) and Voight (1965) analyzed the spacing of fractures in a stiff layer encased in more pliant layers, but their viscous analyses are more appropriate for boudins than for most joints. Sowers (1972) presented a model to explain joint spacing based on the idea that stress concentrations develop at periodic instabilities that form at the interface between layers having different elastic properties. Although Sowers related his model to the spacing of joints, the computations he presented show that extremely high layer-parallel extensional strains are needed to create even very low fracture density. If his model has any applicability to natural fractures it is probably to brittle boudins encased in a viscous matrix, not to joints. Price (1966) suggested qualitatively that spacing of joints is controlled by strain interaction across frictionally coupled bed boundaries.

Hobbs (1967) presented a simple model to explain joint spacing in sedimentary rocks as a consequence of layer-parallel extension, based on the fact that a single joint confined to a layer only releases stress for a short distance along the layer normal to the joint. The rest of the layer remains at a stress close to the fracture stress. Hobbs treated bedded strata as an interlayered elastic sequence, with welded layer boundaries and with different layers having different elastic moduli. We used Hobbs' model because it is based on a reasonably simple but, to first order, physically reasonable view of the interaction between layers containing fractures. Even Sowers (1972) remarked that Hobbs' "explanation may account for fracture spacing in rocks if an instability cannot develop".

Consider Hobbs' model in terms of a single 'jointing' layer between lower-modulus neighbor beds. Some initial joints form at weak points in the layer as a result of a far-field extensional strain. The stress relief that accompanies joint formation is locally damped as a function of the shear modulus, G_n, of the lower modulus neighboring beds. Hobbs assumed that layer parallel shear stress, τ, in a neighboring bed decreases linearly away from the interface with the jointed bed as

$$\tau = \tau_d \left(\frac{T - y}{T} \right), \qquad (1)$$

where τ_d is shear stress at the layer interface, T is thickness of the jointing layer, and y is distance away from the layer interface in the neighboring bed. Tensile stress, σ, along the centerline of the jointed bed increases in magnitude away from a joint in the x direction (parallel to bedding) as:

$$\sigma = E\varepsilon_1 T \left| 1 + \sinh\left(\frac{2}{T}\sqrt{\frac{G_n}{E}}\right) x - \cosh\left(\frac{2}{T}\sqrt{\frac{G_n}{E}}\right) x \right|, \qquad (2)$$

where E and ε_1 are Young's modulus and strain, respectively, in the jointed bed. This equation is derived from Hobbs' equations (6) and (13). From this we see that the magnitude of tensile stress decreases more rapidly away from an existing joint with decreasing T and E, and increasing G_n.

Tensile stress parallel to bedding in the jointed bed increases in magnitude away from an existing joint as a function of extensional strain in the neighboring bed, ε_n, (Hobbs' equation 14).

$$\sigma = E\varepsilon_n T \left[1 - \frac{\cosh\left(\frac{2}{T}\sqrt{\frac{G_n}{E}}\,\frac{S}{2 - x}\right)}{\cosh\left(\frac{S}{T}\sqrt{\frac{G_n}{E}}\right)} \right], \qquad (3)$$

where S is distance between existing joints. Hobbs showed that the maximum tensile stress occurs midway between two existing joints, and that the spacing of joints is proportional to T, $E^{1/2}$ and $G_n^{-1/2}$.

Hobbs predicted that joints can form at any site along a bed provided it is not close to a pre-existing joint, as illustrated by equation (2). Following the formation of a sufficient number of joints at random sites in the bed, subsequent joints develop midway between pre-existing joints as described by equation (3). Figure 6(a) shows the physical situation envisaged by Hobbs. A jointed layer lies between lower modulus beds, with joints forming the ends of the layer, and this stratified package experiences a far-field extensional strain that increases with time (Fig. 6b). The tensile stress is zero across each joint surface; the stress is transmitted in the adjacent lower modulus beds. Figure 6(c) shows the normal stress as a function of distance along the centerline of the jointed bed. The tensile stress is zero at the left-most joint ($x = 0$), increases to a maximum at the midpoint between joints, then symmetrically returns to zero at the joint on the right side. When the magnitude of the tensile stress reaches the tensile strength, C_0, of the layer a new joint forms midway between existing joints, and tensile stress goes to zero at this point. The stress distribution after formation of the new joint at J_1 is shown in Fig. 6(d). Continued extension leads to the formation of new joints at the midpoints, J_2, between existing points, and the stress distribution at this stage is shown in Fig. 6(e).

We tested Hobbs' model of joint spacing in a simulation by placing some initial joints at random locations along a layer. We simulated a joint system by successively breaking each longest joint-bounded segment midway between existing joints. The spacing distribution generated by this model is characteristically multi-peaked (Fig. 7). In contrast, the observed joint spacing distribution in the Monterey Formation is single-peaked, although skewed in the same sense (compare Figs. 4 and 7).

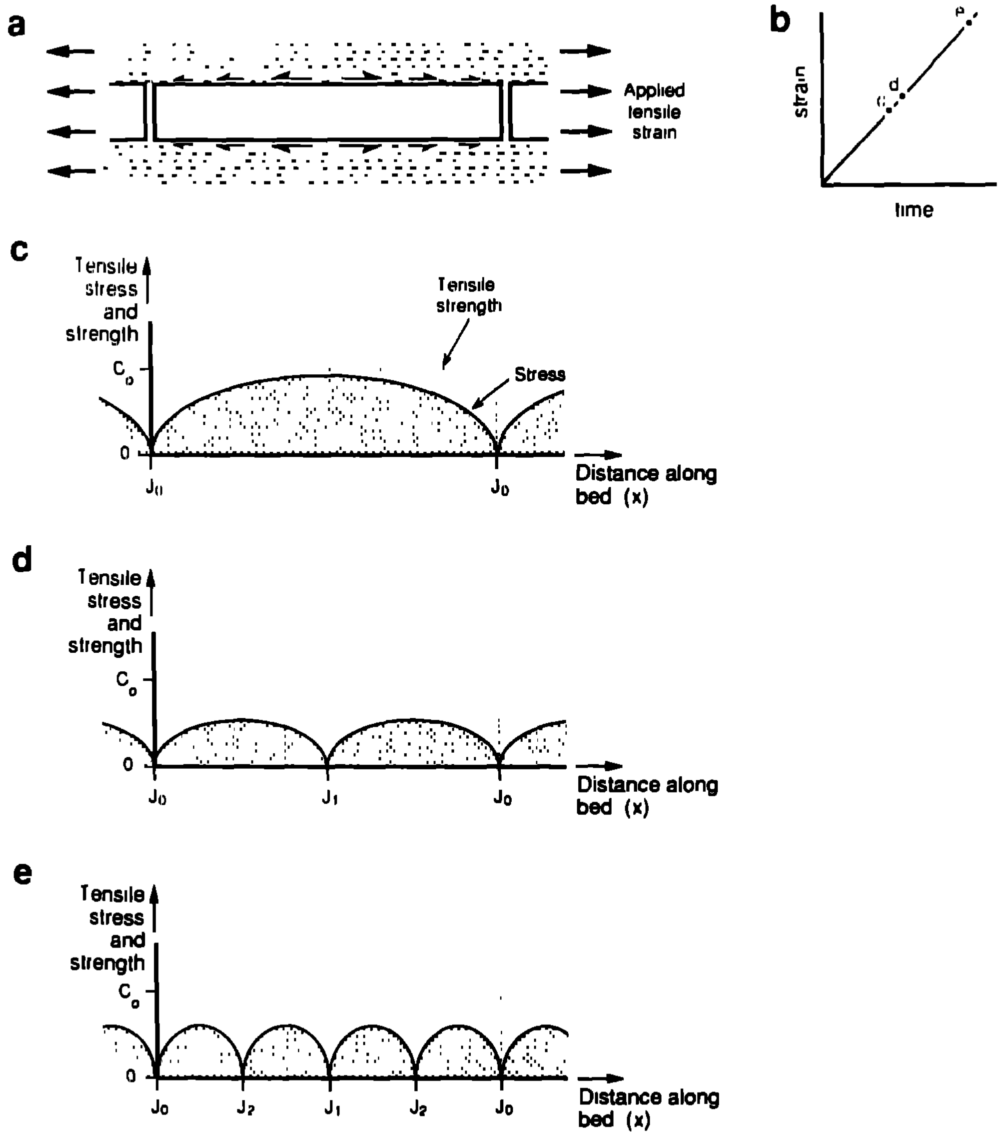

Fig. 6. The Hobbs' model of joint formation. (a) Shear stress along the interface between the higher modulus jointed bed and lower modulus neighboring beds in response to far field extension. (b) Far field extensional strain increases with time. Points c–e are the stages shown below. (c) Tensile stress within the jointed layer vs distance from existing joints J_0 just prior to formation of a new joint, and (d) just after formation of joint J_1. (e) Subsequent joints, J_2, form midway between existing joints.

Addition of flaws to Hobbs' model

A significant aspect of fracture formation that Hobbs did not address is the effect of flaws, which are present in all brittle materials and which form a centerpiece of the science of fracture mechanics (Atkinson 1987). Griffith (1921, 1924) demonstrated that even microscopic cracks magnify stress in a material, effectively reducing its strength. Microcracks may magnify stress, but they are so pervasive in rock that they likely determine its effective tensile strength, and hence do not have any effect on the joint spacing distribution. Joints commonly nucleate from macroscopic flaws such as fossils, concretions and bedding plane irregularities (Engelder 1987, Pollard & Aydin 1988, Kulander *et al.* 1991). These macroscopic flaws may be widely variable in size and widely spaced, in the Monterey Formation they may include fish scales and bones. They will magnify stress more than microcracks because the stress magnification increases by the square of the flaw or crack length (Jaeger & Cook 1979, p. 338).

We made a simple modification of our Hobbs' model simulation by adding flaws that reduce the strength of the jointing layer (Fig. 8). Flaws are placed at random locations on the x axis along the centerline of the bed—in effect we are making a very simple one dimensional model of the effect of flaws. Furthermore the flaws are made to reduce the tensile strength of the layer by randomly varying amounts. In Fig. 8(a) the lines labeled $f_1, f_2, \ldots, f_n$ represent locations and relative sizes of the flaws. Figure 8(b) gives the tensile strength of the jointed bed along the x direction, for example the strength of the bed at flaw f_1 is C_{f_1}, whereas the strength away from the flaws is C_0 and the strength at the joints, J_0, is zero. We still use the stress distribution along the centerline predicted by Hobbs (equation 3), assuming that the flaws do not perturb the stress field to the same order as the joints. Joints are allowed to form wherever the

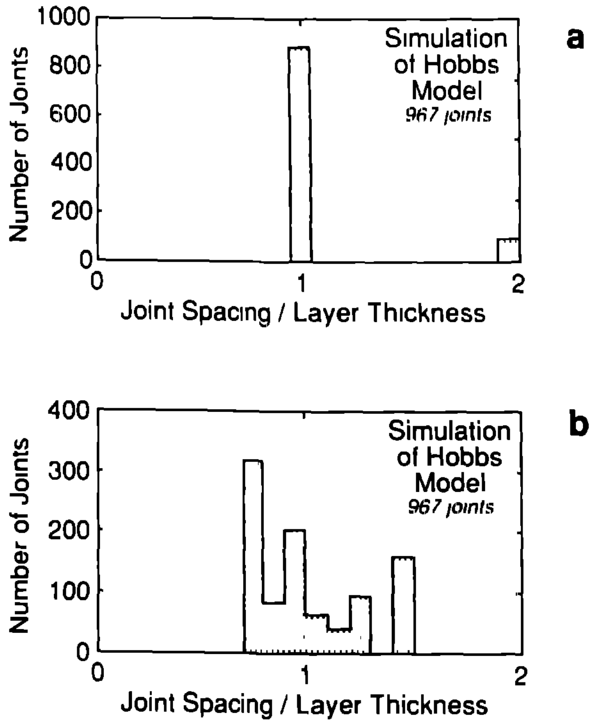

Fig 7 Joint spacing distribution predicted by Hobbs' (1967) model (a) Simulation with primordial joints spaced 1000 times layer thickness (b) Simulation with primordial joints widely and randomly spaced

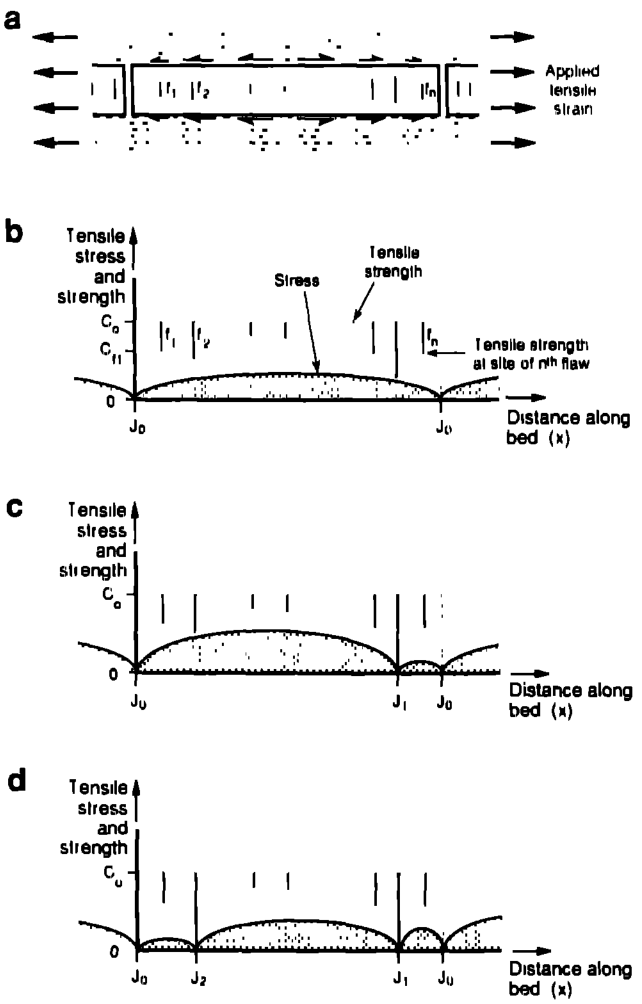

Fig 8 (a) The Hobbs' model of joint formation with flaws randomly located at $f_1, f_2, \ldots, f_n$ (b) Tensile stress vs distance in the jointed layer The amount that a flaw reduces the tensile strength of the jointed layer is represented by the flaw length, so the strength at flaw f_1 is C_{f_1}, etc Subsequent joints form (c) at J_1 and (d) at J_2

tensile stress magnitude equals the local rock strength Although the tensile stress magnitude reaches its maximum value midway between existing joints, the next joint to form is commonly at a flaw, as in Fig 8(c). Most new joints will not be at the midpoint between preexisting joints

The joint spacing distribution of Fig 9 was produced using the flaw modified Hobbs' model This distribution compares well in form with the actual distribution of joint spacing in the Monterey Formation (Fig. 4) In particular it is more nearly single peaked than the model without flaws (Fig. 7) and shows a distribution closer to log-normal. The elastic properties used to generate this model are typical values for a chert or dolostone layer between weak mudstone (Kulhawy 1975, Lama & Vutukure 1978). The results shown in Fig. 9 are from a simulation of 500 joints in a layer whose original length to thickness ratio is 500 1, to produce a simulated mean fracture-spacing index of 1 0. One hundred flaws were assigned to random sites in this simulation, and their effect on tensile strength varies randomly from 0 1 C_0 to C_0.

The simulated joint spacing in Fig. 9 is similar in general form to the actual spacing of joints in the Monterey Formation (Fig. 4), however it is not clear whether it is better described as a linear or a log normal distribution The ratio of the standard deviation of normalized joint spacing to mean joint spacing measured at outcrops is 0.56. In the model simulation of Fig 9 the ratio is 0.39 This similarity between the field data and the results of a simple one dimensional simulation suggests that Hobbs' stress distribution, in the presence of macroscopic flaws that weaken a jointed layer, may be a plausible model of joint spacing in sedimentary strata

Although Hobbs' model with flaws reasonably describes the form of the relationship between layer thickness and joint spacing, the model is not in accord with several important field observations. Two conditions that must be met formally for Hobbs' stress distribution to apply are, first, that thickness of the jointed bed is less than or equal to the the thickness of the lower modulus neighboring bed, and second, that no slip occurs at the interface. It is clear from the field data that the strata we studied violate these assumptions The mechanical layer boundaries are very thin relative to jointed layer thickness, and slip has taken place along these weak layers in some places (this is particularly clear in core of the Point Arguello oil field).

These assumptions affect the form of the stress distribution in the jointed layer. The exact form of the stress distribution is not as important as the general trend predicted by Hobbs; tensile stress is relieved in the region where a new joint forms and tensile stress magnitude increases as a function of distance from the joint. We tested the flaw modified Hobbs' model with various

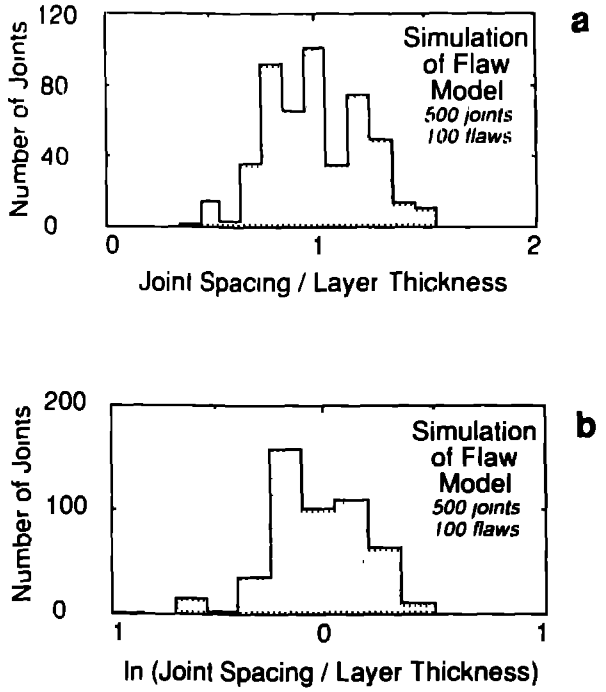

Fig. 9. Typical joint spacing distribution predicted by flaw modified Hobbs' (1967) model. (a) Linear scale and (b) natural log scale. The initial model contained 100 randomly placed flaws that weakened the layer to as little as 10% of its unfractured tensile strength. Other model parameters: E = 66,200 MPa, G_n = 280 MPa, C_0 = 5.0 MPa, starting thickness to spacing ratio = 1.500.

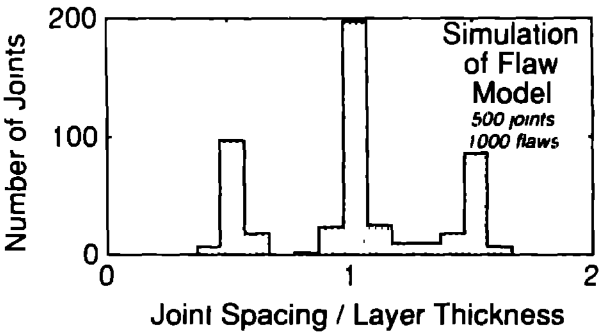

Fig. 10. Joint spacing distribution predicted by flaw modified Hobbs' model. The number of flaws is relatively large. One thousand randomly placed flaws weakened the layer to as little as 10% of its unfractured tensile strength. Other model parameters: E = 66,200 MPa, G_n = 280 MPa, C_0 = 5.0 MPa, starting thickness to spacing ratio = 1.500.

values of Young's modulus of the jointed layer and shear modulus of the neighboring beds. This changes the form of the stress distribution, but the predicted joint spacing distribution is essentially the same as that of Fig. 9.

Changing the coupling between layers from no slip to, for instance, a frictionally-coupled interface will change the stress–distance function, but qualitatively the stress function will have the same basic character of increasing to its maximum magnitude midway between existing joints. This is the basis of the qualitative model of joint spacing proposed by Price (1966).

A further consideration is the limitation of the model to the one-dimensional computation of joint spacing along a line. A one dimensional model may approximately describe the formation of the first formed joint set in a sequence of strata, and so it may be appropriate for comparison with data from the sites in the Monterey Formation we studied, where one joint set clearly predominates. But subsequent joint sets will be affected by a mechanical interaction with pre-existing joint sets as well as by layer boundaries.

One parameter that does affect the form of the joint spacing distribution is the number of flaws relative to the number of joints that are generated. As the number of available flaws approaches and exceeds the final number of joints in these simulated systems, where the ultimate thickness to spacing ratio is unity, the shape of the joint spacing distribution becomes multimodal, as for example in Fig. 10. The form of this distribution, simulated by allowing the number of flaws to equal twice the ultimate number of joints, is rather symmetrical. The peaked shape is suggestive of the idealized Hobbs' model simulations of Fig. 7. Perhaps the effect of a large number of flaws is to reduce the overall strength of the layer and so to allow joints to develop much as they would if no flaws were present. The simulated distributions that look most like the observed joint spacing distributions are obtained when the final joint count is about 4–5 times greater than the number of initial flaws.

In summary, we have shown that it is possible to reasonably model the form of the observed joint spacing distribution by a simple one dimensional simulation of the effect of macroscopic flaws in the jointed layer. Joints are of course a three dimensional phenomenon, more complex than our simple one-dimensional simulation, nevertheless we suggest that modest numbers of macroscopic flaws may be the essence of the observed form of the joint spacing statistics.

DISCUSSION

Voight & St. Pierre (1974), Haxby & Turcotte (1976), Narr & Currie (1982) and Engelder (1985) modeled the evolution of stress in sedimentary strata during a cycle of burial, diagenesis, tectonism, uplift and erosional unloading, and they concluded that strata are most likely to experience horizontal extensional strain during uplift, unroofing and cooling of the basinal sequence. Narr (1991) shows that the density of joints in strata of the Monterey Formation in the deep subsurface (2100–2400 m depth) of the Point Arguello oil field varies (fracture spacing index of 0.08–0.45) with location, but at outcrops the joint density is relatively constant and higher (fracture spacing index of 1.3) over a large area. The diagenetic grade of the siliciclastic rocks we studied at outcrop indicate they reached a temperature of about 100°C, which requires that they were once buried to a depth of about 2.5–3.0 km.

We envisage a joint set beginning to form at depth in strata at the Monterey Formation. While still buried the joint density is sensitive to variations in tectonic strain between different structural positions Here the processes of Hobbs' model operate and the joint density is closely related to extensional strain parallel to bedding As strata are uplifted and extensional strain continues to increase, a condition is reached where it becomes easier to accomplish this stretching by opening existing joints together with sliding along mechanical layer boundaries than by creating new joints At this stage the strata are saturated with joints. The strata that we examined in outcrop, which all show about equal fracture density (fracture spacing index ≈ 1.3) have perhaps reached this saturation level After saturation is reached it is fruitless to compare fracture density among different locations or rock types because subtle differences in fracture density will disappear as less strained strata continued to form joints while more densely jointed rocks strain by opening of existing joints and sliding on mechanical layer boundaries

CONCLUSIONS

At coastal exposures in central California relatively hard, cohesive rocks of the Monterey Formation show a constant ratio of layer thickness to joint spacing of about 1.3 This ratio is called the *fracture spacing index*. It is approximately the same in outcrop among different rock types and in different structural locations over a substantial region (Fig. 1), whereas it is much less—0.08–0.45—in the subsurface of the Point Arguello oil field where the rocks have not undergone uplift and cooling. The frequency distribution of the joint spacing data is log normal. Relatively soft, non cohesive mudstones do not have regular joint sets

Hobbs' (1967) model of the controls on joint spacing, which is based on the idea that joints confined to a layer release stress in the layer close to the joint, qualitatively predicts a constant ratio of layer thickness to joint spacing in an interstratified sequence of rocks with different elastic properties However, a simulation based on this model gives a multimodal spacing frequency distribution The addition of macroscopic flaws, which weaken the jointed bed at random sites along its length, results in a simulated frequency distribution that is similar in form to the observed log-normal distribution

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